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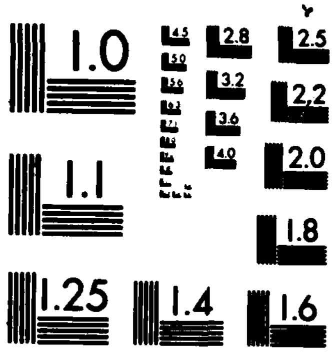


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19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

This research was concerned with analytical results related to control of distributed parameter systems. Existence of optimal controls was established for a variety of systems governed by partial differential equations, most of ~~which~~ involved boundary control. Much of the work also dealt with nonlinearity. The work on "switching systems" promises to provide a theoretical baais for dealing under this grant, including "A class of stabilizing feedback laws for marginally stable systems" answers the question of how costly it is (in terms of control energy) to control quickly.



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# - FINAL REPORT -

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# AFOSR-82-0271

**1982-1986**

**Thomas I. Seidman**



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## FINAL REPORT

In a sense, the Final Report really *is* the attached set of titles and abstracts of research done by the Principal Investigator during the grant period (mid-1982 through 1986). That covers items published since the inception of the grant (excluding 4 publications which appeared in 1982 but were submitted in final form prior to the grant), items accepted or submitted for publication, and items in preparation but sufficiently advanced toward completion as to reasonably ensure that they will eventually appear with credit to AFOSR-82-0271.

No attempt will be made here to catalogue all the conferences attended or colloquium and seminar talks presented on research supported under the grant. The extent to which these have provided stimulation to research — apart from the direct opportunity to hear about others' work — should be clear from the number of conference papers which have subsequently been extended[1] and the number of joint papers.

The abstracts speak for themselves with more detail than need also be provided here. Nevertheless, it seems appropriate to note here, in addition to the cross-references provided in the abstracts, some significant themes.

**1.** A fair number of the papers have been concerned with demonstrating the existence of (optimal) controls for a variety of distributed parameter problems. Most of these have involved boundary control so it became necessary to use methods from functional analysis and modern PDE theory (semigroup theory, trace and embedding theorems, integral equations, Ekeland's theorem, fixed point arguments, etc.) to handle the technical complications involved. Much of the material only began to be of interest at the point where the 'obvious' extensions of finite-dimensional techniques became clearly inadequate. Similarly, a number of the papers[2] were concerned with the justification of 'necessary conditions' for optimality.

**2.** Most of the work here is concerned with issues of nonlinearity. For some (e.g., [2]) this occurs peripherally to the principal considerations of the paper but for others

[1] A case in point is [16]"A one-sided miscellany". The original question, about the characterization of optimal *nonnegative* boundary controls for the heat equation, arose in a conversation at a conference and was duly recorded in a 'questions book'. Some years later, when G. Chen invited me to participate in a special session of the Conference on Engineering Science at PSU, this became the heart of the talk, to which were added some other related items, partly worked out while driving up to PSU with T. Nambu who was visiting UMBC (with support from the grant). With a bit more work, this became my talk at the 4-th IFAC Symposium on DPS last summer. Now, an expanded version of this (in particular, correcting and expanding the abstract form of the main result) is in preparation for publication in 'Automatica'.

[2] [4], [16], [30] - but also including [27] for which the original formulation was not 'variational' but for which the reformulation as an 'optimal control' problem made it possible to show existence of *nontrivial* solutions.

(e.g., [28]) this is the main point — i.e., the extent to which analysis of related linear problems can provide useful information about a nonlinear problem.

**3.** While none of the papers was explicitly computational in nature, a recurrent theme was a concern for 'continuous dependence' — essentially asking whether the results were 'robust' and preparing for consideration of the availability/efficacy of computational approximations[3].

**4.** One development which seems likely to be of considerable continued interest ([30], [37], [42], [46],...) is the work on 'switching systems'. While originally conceived in an attempt to model thermostats, this notion provides a theoretically interesting generalization of 'differential equation' which seems likely to capture the essential aspects of a number of situations involving hysteresis phenomena. One has a situation in which the (nonlinear) dynamics are given by $\dot{x} = f(x, y)$, $\varepsilon\dot{y} = g(x, y)$ and one reduces this to consideration of $\dot{x} = F_k(x)$ where each $F_k(x)$ is $f(x, Y_k(x))$ with $Y_k$ a (stable) solution branch of $g(x, Y) = 0$. It is the stability consideration for the 'fast dynamics' which makes it necessary to be extremely careful in formulating the reduced model. The notion of 'switching system' seems to provide the right setting for this.

**5.** After a gap, there has been some new work on inverse problems (system identification for distributed parameter problems). The continuation of this was stimulated by the coincidence at Oberwolfach last May of conferences on control of PDEs and on inverse problems which both led directly to collaboration with Rundell and to R.S.Anderssen's invitation to visit at Canberra, leading to collaborative work with Vogel and Eldén.

**6.** The interest in semiconductor device modelling led to a sequence of papers ([13], [15], [41], [44], [49]) whose continuation will be funded by AFOSR (under a new grant). So far, these have been primarily analytical but a return to teatment of computational aspects is anticipated although that will closely involve a more detailed analysis of the regularity of solutions, especially near corners.

**7.** Finally, we note an interesting thread of consideration of the problem of the title of [40]: "How violent are fast controls?" That paper answers for the lumped-parameter case a question which had, surprisingly, previously been considered in distributed parameter contexts in [6] and [12]: how costly is it (in terms of control energy required) to control quickly?

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[3] Of interest in this connection is a result in an earlier (conference) paper. In considering boundary nullcontrol for the heat equation, one could proceed by constructing a finite-dimensional approximation *to the model*, obtaining a system of ODEs, and then compute an optimal nullcontrol for that. It can be shown that the results converge (in a not entirely satisfactory sense) to the correct control for the infinite-dimensional problem but that it is easier/better to analyse the distributed parameter problem directly and only go to finite-dimensional approximation later, for computation. *(It is anticipated that this will be expanded for journal publication with the inclusion of relevant comparisons of numerical experiments.)*

**RESEARCH BY T.I. SEIDMAN: 1982-...**
supported under grant #AFOSR-82-0271

1. **State estimation for a Stefan problem,**
   Proc. 21-st IEEE CDC, pp. 1082-1083, IEEE, N.Y., (1982).

   For the one-dimensional, one-phase Stefan problem with unknown initial data, it is shown that observation during $(0, T]$ of the motion of the free boundary suffices to determine the internal state at time $T$ and that this problem is well-posed. The argument is similar to (and, indeed, reduces to) a corresponding result for the ordinary heat equation.

2. **Stabilization of nonlinear parabolic equations,**
   3-rd IFAC Symp. on Control of Distributed Parameter Syst. (preliminary proceedings, pp. IV:6-IV:9), (1982).

   Consider the construction of implementable feedback control laws for stabilization of $\dot{u} + \mathbf{A}u = f(u)$, primarily through boundary control. An abstract result is present and then exemplified by a heat equation $(\mathbf{A} = -\Delta + c)$, using point observation at finitely many interior points.

3. **Existence of optimal controls for some nonlinear distributed parameter systems,**
   Proc. 1983 Conf. on Inf. Sci., Syst., pp. 558-590, Johns Hopkins Univ., (1983).

   One result, here, is existence of an optimal control, minimizing $J = \|u\|_{\Sigma}^2 + \|x(T) - \xi\|_{\Omega}^2$, for the boundary control problem:

   $$\dot{x} - \Delta x = \phi(x), \quad x = u \quad on \quad \Sigma, \quad x(0) = x_0 \quad on \quad \Omega$$

   where we assume $\phi$ is continuous, nonincreasing, and of linear growth with $x_0$ and $\xi$ in $L^2(\Omega)$.

4. **Existence and regularity of extrema,**
J. Math. Anal. Appl. **94**, pp.470-478 (1983).

Suppose one has existence of $x = argmin_{\mathcal{X}} f$ and could obtain regularity (e.g., $x \in \mathcal{Y}$) if $f$ were differentiable on $\mathcal{X}$ so one could justify the 'optimality condition'. If $f$ is differentiable on $\mathcal{Y}$, one can use Ekeland's 'approximate variational principle' to demonstrate the desired regularity.

5. **Boundary observation and control of a vibrating plate,**
in "Control Theory for Distributed Parameter Systems and Applications" (Lect. Notes Control, Inf. Sci. #54; F. Kappel, K. Kunisch, W. Schappacher, eds.), pp. 208-220, Springer-Verlag, Berlin, (1983).

A vibrating plate can be modeled by $u_{tt} + \Delta^2 u = 0$. It is shown, for somewhat special BC, that exact boundary control is possible (in arbitrarily short time). This generalizes to higher dimensions corresponding results by W. Krabs for the vibrating beam. The result is based on new estimates, uniform in a parameter, for a harmonic analysis problem. *See [18] and [43] for generalizations of these estimates.*

6. **Two results on exact boundary control of parabolic equations,**
Appl. Math. Opt. **11**, pp. 145-152 (1984).

It is shown that:
(1) For boundary nullcontrol of a class of parabolic equations, the norm of the control operator grows as $\exp[\mathcal{O}(1/T)]$ as $T \to 0$. (Note that E. Guichal has shown this is sharp.)
(2) One has exact boundary nullcontrollability for certain one-dimensional parabolic equations: $u_t = u_{xx} + q(t, \cdot)u$. This is based on a representation theorem due to Colton and is the first such result for nonautonomous equations. *(Compare the joint work [47] with Rundell.)*

7. **Boundary feedback stabilization of a parabolic equation,** in "Analysis and Optimization of Systems, Part 1" (Lect. Notes Control, Inf. Sci. #62; A. Bensoussan, J.L. Lions, eds.), pp. 385-392, Springer-Verlag, Berlin, (1984).

   A construction is given for 'feedback' stabilization by boundary control for a parabolic equation. The notion of feedback considered involves the construction of a new, stable semigroup acting on an augmented state space corresponding to inclusion of (finite) observation histories. This differs in perspective from 'observer' theories in that the construction is developed from assumed knowledge of (open loop) stabilizing control and asymptotic state estimation. By approximation, the resulting feedback lends itself to a 'pipelining' implementation for real-time (digital) control.

8. **$L^\infty$ bounds for solutions of parametrized elliptic equations,** J. Diff. Eqns. 56, pp. 216-228 (1985).

   A maximum principle argument is used to obtain a differential inequality with respect to a parameter $\lambda$ for elliptic PDEs:

   $$-\nabla \cdot a(\cdot)\nabla u = g(\cdot, u; \lambda)$$

   with suitable $\lambda$-dependent nonlinear BC. This generalizes an earlier result of the author's for $-\Delta u = \lambda f(u)$ with homogeneous Dirichlet conditions.

9. **Will the self-tuning approach work for general cost criteria?** (with P.R. Kumar and W. Lin), Syst. Control Letters 6, pp. 77-85 (1985).

   We expose a fundamental limitation of the 'self-tuning procedure' for adaptive control of linear stochastic systems: estimate the unknown parameter and apply the control input optimal for the estimated parameter. When Ljung's ODE analysis applies to give the asymptotic behavior, it is shown that the system is, indeed, self-tuning to the correct optimal control law *only* if the cost criterion is quadratic (minimum output variance).

10. **A convergent approximation scheme for the inverse Sturm-Liouville problem,**
Inverse Problems **1**, pp. 251-262 (1985).

Convergence (to the correct potential $\psi$) is shown for application of the method of 'generalized interpolation' to the inverse Sturm-Liouville problem:

> Recover $\psi$ from knowledge of the eigenvalue sequence $\{\lambda_k(\psi)\} = \sigma(\mathbf{A}_\psi)$ for the operator $\mathbf{A}_\psi : y \mapsto [-y'' + \psi y]$ (with suitable BC at, say, $-1, 1$).

The principal effort is to define the operator suitably on $L^2(-1,1)$ when $\psi$ is 'rough' (essentially only in $H^{-1}$) and then to show continuity of each $\lambda_k(\psi)$ with this topology for $\psi$; this makes a 'standard' convergence argument for generalized interpolation applicable under a uniqueness assumption (e.g., for $\psi$ symmetric on $[-1,1]$).

11. **A class of nonlinearly elliptic problems,**
J. Diff. Eqns. **60**, pp. 151-173 (1985).

A 'strict coercivity estimate' in $W^{1,p}$ is obtained for a class of nonlinear operators of the form: $\mathbf{A} : u \mapsto -\nabla \cdot a(\cdot, |\nabla u|)\nabla u$ with $a$ of 'power growth' in $|\nabla u|$. This is used to show existence of solutions to the nonlinear elliptic boundary-value problem:

$$-\nabla \cdot a(\cdot, u, |\nabla u|)\nabla u = f(\cdot, u, \nabla u)$$

with (inhomogeneous) Dirichlet conditions.

12. **On boundary controllability of a vibrating plate** (with W. Krabs and G. Leugering),
Appl. Math. Opt. **13**, pp. 205-229 (1985).

Elastic vibrations are modeled, by $u_{tt} + \mathbf{A}^2 u = 0$ where $\mathbf{A}$ is the Laplacian on $\Omega$ with suitable BC. If $\Omega = \Omega_0 \times [0,1]$ (e.g., a rectangular plate), then one has exact boundary controllability for arbitrary time intervals $[0,T]$ Further, one has $\|control\| = \mathcal{O}(T^{-1/2})$ for large $T$ and $exp[\mathcal{O}(1/T)]$ for small $T$; also included are related results on minimum-time control.

13. **Time-dependent solutions of a nonlinear system arising in semiconductor theory** (with G.M. Troianniello),
Nonlinear Anal. -TMA **9**, pp. 1137-1157 (1985).

Semiconductor device physics is modeled by a modified van Roosbroeck system, including recombination and temperature coupling. Assuming constant mobilities, estimates are obtained using maximum principle arguments which then permit application of the Schauder theorem to show existence of solutions and then well-posedness.

14. **Coefficient identification for a parabolic problem,**
*in* "Distributed Parameter Systems" (Lect. Notes Control, Inf. Sci. #75; F. Kappel, K. Kunisch, W. Schappacher, eds.), pp. 340-351, Springer-Verlag, Berlin, (1985).

We consider a parameter identification problem arising in biology. Of principal interest, theoretically, is the use of Ekeland's theorem in arguing the existence of an 'optimal' input, maximizing the local sensitivity to the parameter to be identified. *(An abstract version of the key part of the argument is presented in [19].)*

15. **Time-dependent solutions of a nonlinear system arising in semiconductor theory, II: boundedness and periodicity,**
Nonlinear Anal. -TMA **10**, pp. 491-502 (1986).

Under slightly stronger hypotheses for the same model as [13], it is now shown that the solution will remain uniformly bounded (in time) for correspondingly bounded data. With these estimates it is then shown, assuming periodicity of model and data, that there exists a time-periodic solution.

16. **A one-sided miscellany,**
*in* IFAC 4-th Sympos. on Dist. Param. Syst.(preliminary proceedings), (1986).

We examine the effect, for a number of distributed parameter control problems, of requiring that the controls be non-negative. It is shown that:
(1) The minimum control time for exact boundary control of the one-dimensional wave equation is unaffected by this constraint.

(2) A feasible boundary control for the heat equation having the form: (positive part of optimal control for some *unconstrained* problem) is necessarily the (unique) optimal control. (This generalizes to an abstract result on 'nearest points' to convex sets.)

17. **The method of 'generalized interpolation' for approximate solution of ill-posed problems,**
*in* "Inverse Problems" (J. Cannon, U. Hornung, eds.), pp. 155-161, Birkhäuser, Basel (1986).

This is essentially an exposition of the author's approach to the construction of consistent approximation schemes for ill-posed (inverse) problems:

> Formulate the problem as a sequence of scalar conditions, then approximately minimize a suitable functional (e.g., a norm) subject to approximate imposition of finitely many of these.

It is shown that one has convergence to the correct solution for a class of computationally implementable schemes under quite mild conditions.

18. **The coefficient map for certain exponential sums,**
Proc. Kon. Ned. Akad. Wet. ser. A, **89**, pp. 463-478 (1986).

This generalizes results obtained by the author motivated by application to boundary controllability of a vibrating plate [5], [12]. Consider functions of the form:

$$f(t) = \sum_k c_k e^{i\lambda_k t} \in L^2(-\delta, \delta).$$

The coefficient map $\mathbf{C}_\delta : f \mapsto \mathbf{c} = (c_k)$ is well-defined to $l^2$ for any $\delta > 0$ if $\Lambda = (\lambda_k)$ is quadratically spaced. It is shown here that $log\|\mathbf{C}_\delta\| = \mathcal{O}(1/T)$ as $T \to 0$, with uniformity with respect to certain spacing parameters of $\Lambda$.

19. **Existence for minimization in Banach space with some applications** (with V. Barbu),
J. Math. Anal. Appl. **121**, pp. 96-108 (1987).

Consider a Gâteaux-differentiable coercive lsc functional $f$ (in general not convex) satisfying a structural condition: $f' = \mathrm{G} + \mathbf{R}$ with G 'inverse compact':

$\{x_k\}$ *bounded and* $\{\mathrm{G}(x_k)\}$ *convergent* $\Rightarrow \{x_{k(n)}\}$ *convergent*

and $\mathbf{R}$ compact. Then, with an argument using Ekeland's theorem, it is shown that $f$ attains its minimum subject to finitely many linear inequality constraints. This is illustrated by two examples of distributed parameter optimal control problems. *(See [14]).*

20. **Justification of necessary optimality conditions for certain integral functionals,**
*in* "Optimal Control of Partial Differential Equations, II" (K.-H. Hoffmann, W. Krabs, eds.), pp. 207-218, Birkhauser, Basel (1987).

Consider a functional $F$ of the form: $F(x) = \int_0^T f(t, x(t), z(t))dt$ with $z = \mathrm{G}(x)$. Assume the operator G is 'nice' and that $f$ is 'smooth (in $x$) where finite' but, e.g., $f(t, \cdot)$ may be finite only on a portion of $\mathbb{R}^n$. Assuming existence of a minimum, it is here shown that the *formal* optimality conditions are, indeed, satisfied by the minimizer, typically giving regularity. The argument examines a class of 'special' variations and permits finitely many constraints. *(The original application of this argument was in [27].)*

21. **On certain iterative sequences,**
to appear: Czech. Math J.

In studying 'generalized convergence rates', V. Ptak introduced 'small functions', i.e., $f : [0,1] \to [0,1]$ for which the sequence of iterates is summable for each starting $t \in [0,1]$. It is shown here, along with other related results, that if $f$ is small and continuous and $0 \leq g \leq f$ on $[0,1]$, then $g$ is also small. It is also shown that the set of continuous small functions is closed under *max*.

22. **Dynamics and optimization of a sales-advertising model with population migration** (with N. Derzko and S. Sethi), to appear: J. Opt. Th. Appl.

A somewhat non-standard evolution model is formulated for population distributed over an abstract parameter space and a corresponding sales-advertising model is presented as a distributed parameter optimal control problem. It is shown that the evolution is well-posed and that optimal controls exist for both finite- and infinite-horizon problems. *(Note [24].)*

23. **Stability in the sense of bounded average power** (with P.R. Kumar), to appear: J. Math. Control, Inf.

A sequence $(x_k)$ is 'average $p^{th}$-power bounded' if $\{(\sum_1^N |x_k|^p)/N\}$ is bounded uniformly in $N$; this is reduced to consideration of scalars and $p = 1$. Motivated by an adaptive control problem, we consider classes of systems: $y_{k+1} = a_k y_k + e_k$ with bounds: $0 \leq a_k \leq B$ and $\#\{k \leq K : a_k \geq \theta\} \leq n_K$ for given $B > 1 > \theta > 0$ and, in terms of $B$ and the growth rate of the sequence $(n_K)$, find nasc for the weight sequence $(\alpha_k)$ so the auxiliary condition: "$\{\alpha_k |e_k|\}$ *bounded*" implies that the output sequence $(y_k)$ is average power bounded whenever the input sequence $(e_k)$ is.

24. **Two compactness lemmas**, to appear: *in* "Nonlinear Semigroups, PDE, Attractors" (T.E. Gill, W. W. Zachary, eds.) Springer Lecture Notes in Math.

Two 'tricks' are presented for showing compactness:
(1) Consider a problem for which the initial-value problem is $\mathcal{X}$-Lipschitzian uniformly on bounded sets and (e.g., by use of the Aubin compactness theorem) one has compactness to $L^p(a, b; \mathcal{X})$. Then the initial-value solution map is compact to $\mathcal{X}$. This is useful for applying the Schauder theorem to show existence of periodic solutions. *(See [15], [35].)*
(2) Consider $x' = f(t, x, u), x(0) = 0$ with $f$ uniformly $\mathcal{X}$-Lipschitzian in $x$ and continuous and with $u(t)$ restricted to a compact set. Clearly

the set of control functions $\{u(\cdot)\}$ would not be compact but the corresponding set of solutions $\{x(\cdot;u)\}$ is shown to be precompact in $\mathcal{C}([0,T];\mathcal{X})$. *(This was originally motivated by the argument in [22] for existence of optimal controls.)*

25. **Invariance under nonlinear perturbations of reachable and almost-reachable sets,**
to appear: *in* Proc. IFIP WG-7 Workshop on Control of PDEs, (I. Lasiecka, ed.) Springer-Verlag.

    Let $\mathcal{K}$ be the reachable set (at time $T$) for the control problem:

$$\dot{x} + \mathbf{A}x = \mathbf{B}v, \quad x(0) = 0, \quad v(\cdot) \in \mathcal{V}.$$

    K. Naito introduced, in a special setting, the hypothesis that $\mathcal{K}$ is unchanged when a forcing term $g \in \mathcal{G}$ is added on the right and concluded that it is also unchanged under certain *nonlinear* perturbations. A general, abstract argument is given for this, noting a variety of more concrete settings in which the abstract hypotheses are verifiable. In a special case one can similarly consider the approximately reachable set. *(More detail is given in [28], [36].)*

26. **A class of stabilizing feedback laws for marginally stable systems** (with T. Nambu and R. Rostamian),
to appear: Applicable Analysis.

    Integral equation methods are used to study a class of feedback controls for exponential stabilization of the (marginally stable) standard heat equation in a region with insulated boundary. An example shows that seemingly reasonable feedbacks may lead to instabilities if the gain factor is too large but there is a stable range with an estimable upper bound. Some generalizations and a conjecture are discussed. *This and [29] are work done during Nambu's visit to UMBC, funded by the grant.*

27. **Equilibrium states of a nonlinearly elastic conducting rod in a magnetic field** (with P. Wolfe),
to appear: Arch. Rat. Mech. Anal.

We consider a hyperelastic Cosserat rod with clamped ends conducting current in a magnetic field. The stresses can then be viewed as energy-minimizing 'controls' subject to the boundary conditions as 'state constraints'. The integrand of the energy functional blows up at 'infinite compression' so there are difficulties in justifying the (formal) Euler equations which determine the configuration. The argument for this justification is the principal theoretical novelty of the paper. It can then be shown that for large currents there must exist nontrivial steady-state solutions.

28. **Invariance of the reachable set under nonlinear perturbations**,
to appear: SIAM J. Control and Optim.

Suppose the reachable set $K_g$ for the control problem:

$$\dot{x} + \mathbf{A}x = g + \mathbf{B}v, \quad x(0) = 0, \quad v(\cdot) \in \mathcal{V} = L^p(0,T;V)$$

is the same for all $g \in \mathcal{G} = L^{p'}(0,T;Z)$. Under hypotheses ensuring that the nonlinear problem:

$$\dot{x} + \mathbf{A}x = f(\cdot,x) + \mathbf{B}v, \quad x(0) = 0$$

gives $f(\cdot,x(\cdot)) \in \mathcal{G}$ for $v(\cdot) \in \mathcal{V}$ together with a compactness condition, a fixed point argument shows the reachable set is also invariant under the nonlinear perturbation. The relevant compactness condition is verified under several more concrete hypotheses, including two which may permit applicability to control for hyperbolic PDEs.

29. **Feedback control for an abstract parabolic equation** (with T. Nambu and R. Rostamian),
to appear: *in* Proc. Intl. Conf. on DEs and Math. Phys.

This generalizes the considerations and arguments of [26] to an abstract setting involving semigroups and mildly singular integral equations.

30. **Control-theoretic aspects of switching systems,**
to appear: in Proc. 1987 Conf. on Inf. Sci., Syst., Johns Hopkins,

'Switching systems' (cf., [37], [46]) can, e.g., arise from 'implementation'of certain (formal) feedback policies for multimodal systems. The control-theoretic aspects under consideration here, however, are the implications of discontinuities due to (unmodeled) bifurcations – specifically, the characterization of quasi-optimal control policies.

31. **A proposed generation process for the reconstruction of line drawings** (with C.-H. Lee and A. Rosenfeld),
submitted to: Artificial Intelligence.

The 3-D interpration of 2-D visual data is a standard problem of computer vision, often resolved by comparing the data with some family of possibilities for parametric optimization. Here, a two-step sequential interpretation model (consisting of a generation process followed by a decision process) is proposed. The 'generation process' specifies a 3-D interpretation (implicitly) in terms of the 2-D data and some auxiliary parameters. While more generally applicable, the approach is presented here for the interpretation of line drawings.

32. **The linear complementarity problem for general cones** (with M. S. Gowda),
submitted to: Math. Programming.

Let $K$ be a cone in $\mathbb{R}^n$ and $\mathbf{T}$ *copositive plus* on $K$ (i.e., $\langle \mathbf{T}x, x\rangle \geq 0$ on $K$ with equality only if $(\mathbf{T}+\mathbf{T}^*)x = 0$). C. Lemke showed (for $K = \mathbb{R}^n$) that existence of a feasible $x$ implies solvability of the linear complementarity problem:

> Given $q$, find $x_* \in K$ such that $\langle \mathbf{T}x_* + q, x\rangle \geq 0$ on $K$ with equality for $x = x_*$.

We generalize this to show that the implication holds if and only if $K$ is *polyhedral* and also give a perturbation result.

33. **Periodic solutions of a class of nonlinear parabolic equations**, submitted to: Nonlinear Anal. -TMA.

The Glicksberg fixpoint theorem for multi-valued maps is used to show existence of time-periodic solutions for parabolic equations of the form:

$$u_t - \nabla \cdot a(\cdot, u, |\nabla u|)\nabla u = f(\cdot, u, \nabla u)$$

with Neumann BC: $a\nabla u \cdot \mathbf{n} = \phi$. Suitable hypotheses are imposed to ensure satisfiability of the consistency condition: $\int f = \int \phi$ and applicability of a coercivity estimate from [11].

34. **Continuous dependence for a degenerate parabolic equation**, submitted to: J. Diff. Eqns.

Consider the parabolic equation: $u_t - \nabla a(\cdot, |\nabla u|)\nabla u = f$ with Neumann BC: $a\nabla u \cdot \mathbf{n} = \phi$ and time-periodicity. If $a(\cdot, 0) = 0$, then the equation is degenerate; nevertheless, in a suitable setting one has uniqueness and continuous dependence: not only on the data $f, \phi$ but also on the form of the nonlinearity $a(\cdots)$.

35. **Periodic solutions of a parabolic quasi-variational inequality from stochastic optimal control** (with S. Belbas), submitted to: SIAM J. Math. Anal.

The Hamilton-Jacobi-Bellman approach to optimal control yields, for a time-periodic stochastic control problem with the option of voluntary termination, a periodic parabolic variational inequality of evolution:

$$u \in \mathcal{V} = \{v \in L^2([0,T] \to H^1(\Omega) : v \le \phi \quad on \quad \mathcal{Q}, \quad v = \hat{u} \quad on \quad \Sigma\},$$

$$\langle \dot{u} + \mathbf{L}u - f(\cdot, u, \nabla u), u - v\rangle \le 0 \quad \forall v \in \mathcal{V}.$$

It is shown that this has a unique periodic solution.

36. **Invariance of almost-reachable sets under nonlinear perturbation (with K. Naito),**
submitted to: SIAM J. Cont. Opt.

Define a linear operator $\mathbf{L} : v \mapsto x$ and a nonlinear operator $\mathbf{G} : v \mapsto \mathbf{F}(x)$ by solving (1)$\dot{x} = \mathbf{A}x + v$ and (2)$\dot{x} = \mathbf{A}x + \mathbf{F}(x) + v$, respectively, so $\mathbf{G}(v)$ is the fixpoint of $x \mapsto \mathbf{L}(v + \mathbf{F}(x))$; let $\mathbf{E}$ give evaluation at $T$ and set

$$\mathcal{K}_g = \{\mathbf{EL}(g + u) : u \in \mathcal{U}_{ad}\}.$$

The arguments of [28] "Invariance of the reachable set under nonlinear perturbations" are combined with the abstract theory of interpolation of Banach spaces to analyze the relation of (closures of) $\mathcal{K}_{\mathbf{F}}$ and of $\mathcal{K}_g$ for $g \in \mathcal{G}$. *(Compare [25].)*

37. **Switching systems, I,**
submitted to: Nonlinear Anal. -TMA.

'Switching systems' are generalisations of differential equations: one has several 'modes' (corresponding to evolution given by DEs) with given sets in the state space on which transition from one mode to another is either permitted or mandatory. The theory presented here was motivated by modeling of a thermostat but such systems arise also in certain control-theoretic contexts and for reduced order models with unmodeled bifurcations. This paper presents basic existence/continuity theory and examples illustrating some possibly surprising phenomena.

## WORK SUBSTANTIALLY IN PROGRESS

{ (T) typing or (W) writing already in process }

*A variety of other work is also in progress at a variety of stages of development. Included here are only those papers which have already reached a point (semicompletion or, at least, substantial results already obtained) at which it is reasonably certain that they will, indeed, appear with credit to the grant.*

38. (T)**An inverse eigenvalue problem with rotational symmetry.**

    This adapts the methods of [10] ("A convergent approximation scheme for the inverse Sturm-Liouville problem") to consideration of partial differential operators of the form: $\mathbf{A}_\psi : u \mapsto (-\nabla \cdot a\nabla u + \psi u)$ on the (unit) ball $\Omega$ in $\mathbb{R}^n$ with 'radial' BC and with $a = a(r)$ 'nice' and $\psi = \psi(r)$ 'rough' (essentially as $H^{-1}$, modified near $r = 0$). As in the earlier paper [10], much of the effort is in defining $\mathbf{A}_\psi$ so as to be self-adjoint with compact resolvent on $L^2(\Omega)$ and in showing continuous dependence on $\psi$ for the eigenvalues. Convergence of the method of 'generalized interpolation' (cf., [17]) is then assured for the inverse eigenvalue problem under the assumption of uniqueness.

39. (T)**Well-posedness and convergence of some regularization methods for nonlinear ill-posed problems** (with C. Vogel).

    Two popular approaches to the construction of approximation schemes for ill-posed problems are Tikhonov regularization and 'constrained least squares'. Previous proofs of convergence (to the correct solution) had been given for application of these methods to linear problems. Here, convergence proofs are given in a quite general nonlinear setting. *(It is expected that an additional section will be added with examples - e.g., identifying a coefficient with a jump discontinuity at some unspecified point.)*

40. **(T)How violent are fast controls?**

Consider the standard LQ optimal control problem:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u, \quad x(0) = 0, \quad x(T) = \xi, \quad \|u(\cdot)\| = \min$$

where the norm is for $u \in L^2(0,T;\mathbb{R}^m)$. Let $\mathbf{C}_T : \xi \mapsto u$ be the 'optimal control operator'; clearly, $\|\mathbf{C}_T\| \to \infty$ as $T \to 0$. The present result is that $\|\mathbf{C}_T\| \sim \gamma T^{-(K+1/2)}$ where K is the minimal exponent giving the rank condition ($range[\mathbf{B}, \ldots, \mathbf{A}^K\mathbf{B}] = \mathbb{R}^n$) for controllability and $\gamma$ is computable. *(A graduate student, P. Wang (partially supported by the grant), was of assistance in the formulation of the result for non-scalar controls.)*

41. **(W)Time-dependent solutions of a diffusion-reaction system with electrostatic convection and generation, I.**

Consider a modified van Roosbroeck system for time-dependent semiconductor device modeling. The model includes temperature coupling, recombination, generation terms corresponding to impact ionization, and velocity saturation. (Field-dependent diffusion and corner effects are not treated.) The problem is shown to be globally well-posed under minimal regularity assumptions with some additional continuity/regularity properties. The principal technical difficulties arise from the structure of the generation terms included. *(This has now been separated from [44] for clarity of exposition.)*

42. **(W)Switching systems, II: periodicity.**

Questions of the existence of periodic solutions for switching systems (cf., [37] "Switching systems, I") are complicated by the admissibility of certain nonuniquenesses of evolution. The principle result is construction of an example giving non-existence of any periodic solutions for an autonomous bimodal switching system with a compact, convex, attractive invariant set in $\mathbb{R}^2$. There are also some positive results. *(This work actually appeared with [37] in an earlier version but was separated at the referee's suggestion.)*

43. (W) **The coefficient map for certain exponential sums, II** (with M. Gowda).

Consider functions of the form: $f(t) = \sum c_k exp[i\lambda_k t]$ in $L^2(-\delta,\delta)$ where $\Lambda = (\lambda_k)$ is spaced, roughly, like $[Dk]^p (p > 1)$ for $k = 1, 2, \ldots$ and $\lambda_{-k} = -\lambda_k$. The paper uses much the same arguments as for the previous paper [18] (considering the case $p = 2$) to show that $log\|\mathbf{C}_T\| = O(1/T)$, with certain uniform estimates, where $\mathbf{C}_T = \mathbf{C}_T(\Lambda)$ is the 'coefficient map': $f \mapsto \mathbf{c} = (c_k) \in l^2$. *(Writing is in progress while efforts continue to work out the asymptotics of the estimate in $O(1/T)$ as $p \to 1, \infty$.)*

44. **Time-dependent solutions of a diffusion-reaction system with electrostatic convection and generation, II: field-dependent diffusion.**

This considers much the same model as for the previous paper [41] except for the treatment, here, of field-dependent diffusion – for which the requisite estimates are far more delicate. Using the previous result, a fixed point argument gives existence of a global solution. *(It may be possible also to obtain and include estimates giving uniqueness and continuous dependence under slightly stronger hypotheses.)*

45. (W) **Some problems of distributed parameter control with positivity constraints.**

*This is just a somewhat expanded version (for Automatica) of the earlier conference paper [16] "A one-sided miscellany".*

46. **Switching systems, III: singular perturbation and thermostat models** (with E. Abed).

The notion of 'switching systems' (cf., [37]) was originally motivated by the desire to model thermostats and certain similar devices with (hysteretic) discontinuities of evolutionary mode; a discussion of this is presented here with comparison with some other models. Of perhaps even greater interest is the interpretation of this notion as providing a reduced order model for situations in which an unmodeled singular perturbation leads, at a bifurcation point, to a rapid transient to a non-contiguous stable branch – observable as an apparently discontinuous 'switch' in behavior. *(The thermostat modelling is substantially complete but not as much has yet been done about the singular perturbation aspect as we wish.)*

47. **A semigroup approach to Colton's kernel with application to boundary control and system identification for time-dependent one-dimensional parabolic equations** (with W. Rundell).

D. Colton introduced an integral transformation relating solutions of $u_t = u_{xx} + q(t,x)u$ and of $v_t = v_{xx}$. The kernel $k(t,x,y)$ is a solution of the rather unusual PDE:

$$w_t = w_{xx} - w_{yy} + q(t,x)w$$

with $w = 0$ at $y = 0$ and an inhomogeneous Dirichlet BC at $x = y$ but imposing no initial condition with respect to $t$. A semigroup (acting on a novel space involving conditions on the support of the Fourier transform) is constructed and used to show existence of solutions for more general $q(\cdot)$ than those (analytic in $t$) considered by Colton. This greater generality is needed for the applications presented. *(The applications are related to those of [6] and some work by Rundell and J. Cannon on coefficient identification. These clearly follow from the availability of the representation for more general $q(\cdot)$ but details must still be worked out.)*

48. **An improved hyperbolic approximation for the sideways heat equation (with L. Eldén).**

Suppose it is known that $u$ satisfies the heat equation $u_t = u_{xx}$ for $t, x > 0$ with $u = 0$ at $t = 0$ and $u(t, 0) = \phi(t)$. Suppose, however, that $\phi$ itself is unknown except for an $L^2(0, \infty)$ bound: $\|\phi\| \leq M$. Given the function $g$, corresponding to observation of $u$ at $x = 1$ with $L^2$ accuracy $\varepsilon$ (i.e., $\|g - u(\cdot, 1)\| \leq \varepsilon$), one wishes to reconstruct $u$ with 'optimal' accuracy: a 'log-convexity' estimate gives uncertainty $2M^{1-x}\varepsilon^x$ and one wishes to do 'not much worse' computationally. Error estimates have been obtained by Eldén for the use of an approximation

$$\mu v_{tt} + v_t = v_{xx}, \quad v(0, \cdot) = 0, \quad v(\cdot, 1) = g$$

with the constant $\mu > 0$ suitably chosen, depending on $\varepsilon/M$. The approach here is to take $\mu$ varying in $x$, depending on the error estimate $\varepsilon(x)$, so as to optimize the estimate. *The results so far improve Eldén's earlier estimates but still do not, as hoped, give the log convexity estimate to within a factor; work continues (using BITNET and TEXfor U.S.-Swedish communication).*

49. **Time-dependent solutions of a diffusion-reaction system with electrostatic convection and generation, III: numerical approximation.**

The approach and estimates of [41 and [44] ("Time-dependent solutions of a diffusion-reaction system with electrostatic convection and generation, I and II") are adapted to give error estimates (convergence rates) for computational approximations to a modified van Roosbroeck system modelling a semiconductor device with impact ionization.

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